Higher Prelim Paper 1 2015 **Marking Scheme**

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	**ans: proof** **(3 marks)**	
	●[1] process – synthetic division for example	●[1] $\begin{array}{c\|cccc} 2 & 2 & 3 & -29 & 30 \\ \hline & & & & \end{array}$
	●[2] completes synthetic division	●[2] $\begin{array}{c\|cccc} 2 & 2 & 3 & -29 & 30 \\ & & 4 & 14 & -30 \\ \hline & 2 & 7 & -15 & 0 \end{array}$
	●[3] conclusion	●[3] since remainder = 0, $(x-2)$ is a factor
(b)	**ans: $(x-2)(2x-3)(x+5)$** **(2 marks)**	
	●[1] finds quotient	●[1] $(x-2)(2x^2+7x-15)$
	●[2] factorises fully	●[2] $(x-2)(2x-3)(x+5)$ [must include $(x-2)$]
2	**ans:** $x^2+\frac{1}{x^2}$ **(3 marks)**	**Pegasys Extension Test 2010-11 Q10**
	●[1] substitutes	●[1] $(x-\frac{1}{x})^2+2$
	●[2] removes brackets	●[2] $x^2-2+\frac{1}{x^2}+2$
	●[3] states answer	●[3] $x^2+\frac{1}{x^2}$
3	**ans: $7y-3x+13=0$** **(3 marks)**	
	●[1] finds gradient of given line	●[1] $m=-\frac{7}{3}$
	●[2] finds perpendicular gradient	●[2] $m_{perp}=\frac{3}{7}$
	●[3] substitutes into equation and rearranges	●[3] $y+1=\frac{3}{7}(x-2)$
4	**ans: 1/8** **(3 marks)**	
	●[1] prepares to differentiate	●[1] $f(x)=\frac{1}{2}x^{-2}$
	●[2] differentiates	●[2] $f'(x)=-1x^{-3}=-\frac{1}{x^3}$
	●[3] subs and evaluates	●[3] $f'(-2)=-\frac{1}{(-2)^3}=\frac{1}{8}$

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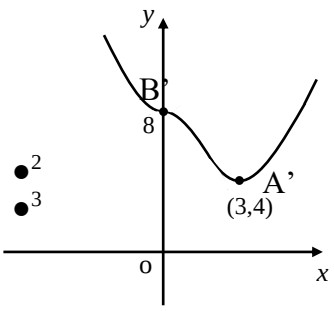
5	**ans:** C_2 or second circle **(5 marks)** $\bullet^1$ state centre of C_1 $\bullet^2$ equates x coordinates to find k $\bullet^3$ finds radius of C_1 $\bullet^4$ uses radius formula for R_2 $\bullet^5$ find R_2 and compare with R_1	**SQA 2006 Higher P2 Question 4** $\bullet^1$ $C_1 = (3, 4)$ $\bullet^2$ $k = 6$ $\bullet^3$ $R_1 = 5$ $\bullet^4$ $R_2 = \sqrt{(-3)^2 + (-4)^2 + 12}$ or equivalent $\bullet^5$ $\sqrt{37} > 5$ or C_2
6	**ans: graph drawn** **(3 marks)** $\bullet^1$ correct shape $\bullet^2$ correct image for A annotated $\bullet^3$ correct image for B annotated	$\bullet^1$ $\bullet^2$ $\bullet^3$ 
7	**ans: 64** **(3 marks)** $\bullet^1$ integrates $\bullet^2$ subs values $\bullet^3$ evaluates	$\bullet^1$ $x^4 - 2x^2$ $\bullet^2$ $[3^4 - 2(3)^2] - [(-1)^4 - 2(-1)^2]$ $\bullet^3$ 64
8	**ans:** $p \le -\frac{2}{9}$ **,** $p \ge 2$ **(4 marks)** $\bullet^1$ knows condition for real roots $\bullet^2$ calculates discriminant $\bullet^3$ strategy for solving $\bullet^4$ factorising to answers	$\bullet^1$ $b^2 - 4ac \ge 0$ for real roots $\bullet^2$ $(-3p)^2 - (4(4p+1).1) \ge 0$; $9p^2 - 16p - 4 \ge 0$ $\bullet^3$ diagram drawn $\bullet^4$ $(9p + 2)(p - 2) = 0 \Rightarrow p \le -\frac{2}{9}$ or $p \ge 2$
9	**ans:** $a = -4$ **(2 marks)** $\bullet^1$ differentiates and equates to 0 $\bullet^2$ subs and solves for a	$\bullet^1$ $2x + a = 0$ $\bullet^2$ $2(2) + a = 0$; $a = -4$

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10(a)	**ans: $y = 2x - 1$ (3 marks)**	**SQA 2006 Higher P1 Question1**
	•1 finds coordinates of D	•1 $D = (3,5)$
	•2 finds gradient of BD	•2 $m_{BD} = \frac{5+5}{3+2} = 2$
	•3 states equation of BD	•3 $y - 5 = 2(x - 3)$ or equivalent
(b)	**ans: $y = -3x + 9$ (3 marks)**	
	•1 finds gradient of BC	•1 $m_{BC} = \frac{-2+5}{7+2} = \frac{1}{3}$
	•2 finds perpendicular gradient	•2 $m_{\perp} = -3$
	•3 finds equation of BC	•3 $y - 12 = -3(x + 1)$ or equivalent
10(c)	**ans: (2,3) (3 marks)**	
	•1 starts solving system of equations	•1 $2x - 1 = -3x + 9$ or equivalent
	•2 Finds value of x	•2 $x = 2$
	•3 Finds value of y	•3 $y = 3$
11	**ans: $p = 0{\cdot}25$ (4 marks)**	
	•1 set up one equation	•1 $12 = 20p + q$
	•2 set up second equation	•2 $10 = 12p + q$
	•3 solve for one variable	•3 $p = 0.25$
	•3 solve for second variable	•4 $q = 7$
12(a)	**ans: proof (4 marks)**	**SQA 2005 Higher P2 Question 2**
	•1 interpret diagram	•1 $\cos p = \frac{8}{17}$ $\sin p = \frac{15}{17}$ stated or implied by •4
	•2 interpret diagram	•2 $\cos q = \frac{8}{10}$ $\sin q = \frac{6}{10}$ same order as •3
	•3 expand sin (A + B)	•3 $\sin p \cos q + \cos p \sin q$ explicitly stated
	•4 substitute and complete	•4 $\frac{15}{17} \times \frac{8}{10} + \frac{8}{17} \times \frac{6}{10} = \frac{120}{170} + \frac{48}{170} = \frac{168}{170} = \frac{84}{85}$
(b)	**ans: $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (3 marks)**	
	•1 any expression equivalent to $\sin 75^0$	•1 $\sin(45 + 30)^\circ$ or equivalent
	•2 correct exact values	•2 $\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$
	•3 correct answer	•3 $\frac{\sqrt{3}+1}{2\sqrt{2}}$

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13(a)	**ans**: $a = 2,\ b = 1,\ c = -5$ **(3 marks)**	**SQA 2006 Higher P1 Question 8**
	•[1] find a •[2] find b •[3] find c	•[1] $a = 2$ •[2] $b = 1$ •[3] $c = -5$
(b)	**ans:** (-1,-5) **(1 mark)**	
	•[1] interpret equation of parabola	•[1] (-1,-5)
14	**ans**: **proof** **(5 marks)**	**SQA 2005 Higher P2 Question 8**
	•[1] equate for intersection •[2] use double angle formula •[3] factorise •[4] process two solutions •[5] complete proof	•[1] $k\sin 2x = \sin x$ •[2] $k \times 2\sin x \cos x$ •[3] $\sin x(2k\cos x - 1)$ •[4] $\sin x = 0 \quad \cos x = \frac{1}{2k}$ •[4] $\sin x = 0 \quad x = 0, \pi, 2\pi$ ie at O,B and D $\cos x = \frac{1}{2k}$ for A and C
		Total: 60 marks