

Harris Academy

Mathematics
Higher Prelim Examination 2015
Paper 2
Assessing Units 1, 2 and 3

**NATIONAL
QUALIFICATIONS**

Time allowed - 1 hour 30 minutes

Read carefully

- 1. Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b} .

or

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \text{where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

ALL questions should be attempted

1. Prove that the line $y = 3 - x$ is a tangent to the circle with equation $x^2 + y^2 - 2x - 12y + 29 = 0$ and find the point of contact. 5

2. Solve algebraically the equation $2 \cos 2x + 8 \sin x - 5 = 0$ in the interval $0 \leq x \leq 2\pi$. 6

3. A scientist studying a large colony of bats in a cave has noticed that the fall in the population over a number of years has followed the recurrence relation $U_{n+1} = 0.75U_n + 200$, where n is the time in years and 200 is the average number of bats born each year during a concentrated breeding week.

(a) He estimates the colony size at present to be 2100 bats with the breeding week just over. Calculate the estimated bat population in 2 years time immediately **before** that years breeding week. 2

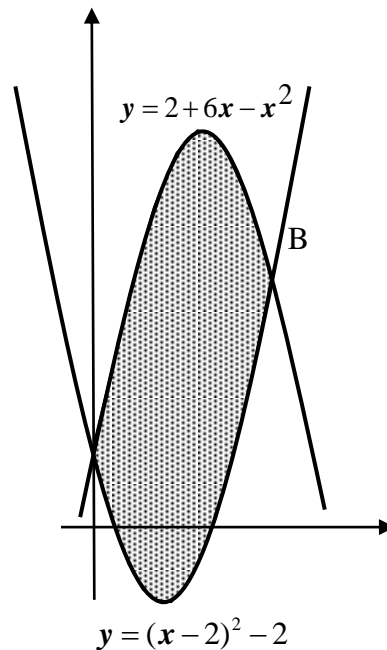
(b) The scientist knows that if in the **long term** the colony drops, at any time, below 700 individuals it is in serious trouble and will probably be unable to sustain itself. Is this colony in danger of extinction? **Explain your answer with words and appropriate working.** 4

4. Given that $f(x) = \frac{x^3 - 3}{\sqrt{x}}$; $x > 0$, find $f'(x)$. 4

5. Two curves with equations $y = (x - 2)^2 - 2$ and $y = 2 + 6x - x^2$ meet at A and B as shown in the diagram.

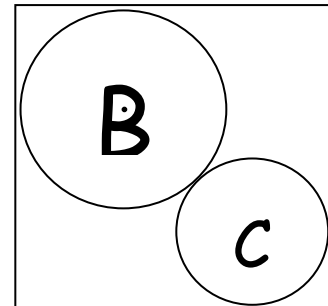
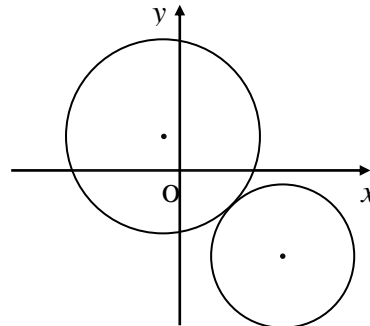
(a) Calculate the coordinates of A and B. 4

(b) Find the area between the two curves. i.e the shaded area in the diagram. 4



6. The logo for a bowling club is as shown in the diagram.

Relative to suitable axes the equation of the larger circle is $x^2 + y^2 + 2x - 4y - 27 = 0$ and the smaller circle has equation $x^2 + y^2 - 12x + 10y + 43 = 0$.



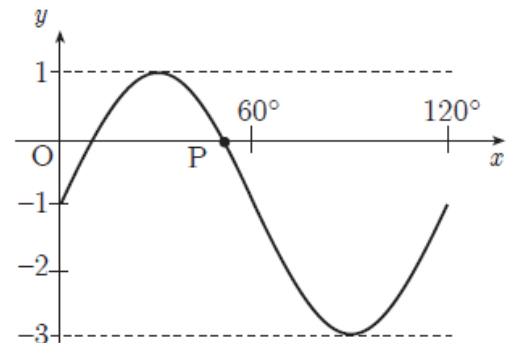
(a) Show that the two circles touch externally at a single point and find the point of contact. 6

(b) Establish the equation of the common tangent at this point. 3

7. The diagram shows part of the graph of a function whose equation is of the form $y = a \sin(bx^\circ) + c$.

(a) Write down the values of a , b and c . 3

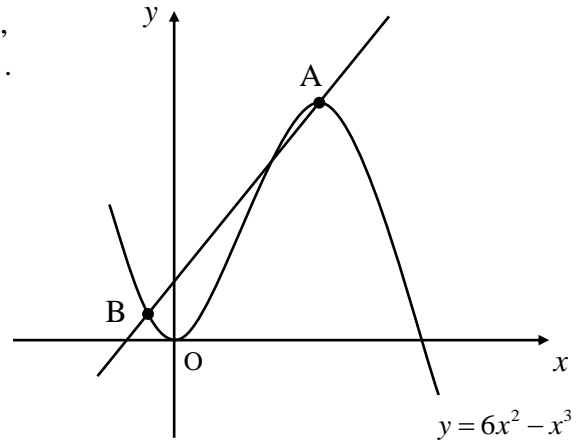
(b) Determine the exact value of the x -coordinate of P , the point where the graph intersects the x -axis as shown in the diagram. 3



8. A curve has as its derivative $\frac{dy}{dx} = 4x - 6x^2$.

Given that the point $(-1, 9)$ lies on this curve, express y in terms of x . 4

9. The diagram below, **which is not drawn to scale**, shows part of the graph of the curve $y = 6x^2 - x^3$.

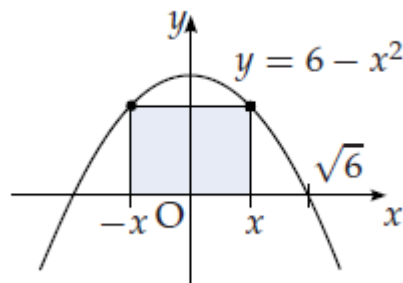


- (a) Find the coordinates of the point A, the maximum turning point of this curve. 4
- (b) The line through A, with gradient 5, intersects the curve at a further two points, one of which is B.

Find **algebraically** the coordinates of B.

Your answer must be accompanied with the appropriate working. 5

10. A rectangle is formed under the graph of $y = 6 - x^2$, as shown in the diagram.



- (a) Show that the area A of the rectangle is given by

$$A(x) = 12x - 2x^3$$

3

- (b) Hence find the value of x which maximises the area of the rectangle, and the corresponding area. 5

11. The line $y = 5x + k$, where k is a constant, is a tangent to the parabola $y = 2x^2 + x - 5$. Find the value of k . 5